

# Stimulating Mathematical Thinking

Innovations in  
Common Core  
math teaching

and their  
implications for  
education reform

## A White Paper

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Evaluation of the Illinois Mathematics and  
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On behalf of

**EVALSOLUTIONS INC.**

For

**Illinois State Board of Education**

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# Stimulating Mathematical Thinking

*This white paper explores the importance of stimulating mathematical thinking, and how new approaches to professional development, curriculum and assessment are helping states and districts achieve this paradigm change in mathematics education in the United States.*

## Executive Summary

The U. S. continues to lag behind other nations on international math assessments such as the PISA tests for high school students. This has serious implications for America's global competitiveness. The Common Core State Standards for Mathematics align well with standards in high-achieving nations. OECD believes their successful implementation will yield significant performance gains on PISA tests. The U. S. Department of Education's Mathematics and Science Partnerships Program (MSP) is the key federal and state level initiative for improving math education in U. S. schools. It is part of the solution to the challenge of implementing Common Core math across the nation.

Twenty-five years after Deborah Ball's seminal research, curriculum development, professional development and assessment for math instruction is shifting to focus on stimulating mathematical thinking and building mathematical skillfulness under Common Core math. Its eight mathematical practices and narrower content focus should promote deeper understanding and skill.

States and national providers are assisting districts in reviewing instructional content for Common Core math alignment. Many districts are also using state and national resources to develop their own Common Core-aligned curricula. A number of Common Core states offer online curriculum toolsets.

Nationwide, district teachers are providing much of the professional development for Common Core math, with universities and professional in the field also making important training contributions. Professional development is focused on increasing

both subject matter and pedagogical content knowledge. Intensive, long-term professional development appears most effective in achieving change in teacher practice.

About half of U. S. states have replaced existing state assessments with one of the assessment systems developed by two state-led consortia. Both of the new assessment systems combine formative and summative assessment features. States face multiple barriers to implementing these new assessment systems. Embedding formative assessment activities such as student performance tasks into Common Core math curriculum and professional development is a way of stimulating student mathematical thinking and practice and preparing for Common Core state assessments.

Multiple education reforms are being carried out today in school across the U. S., include Common Core State Standards (CCSS) and associated curriculum, instructional and assessment reforms; the PARCC/Smarter Balanced common sets of state assessments aligned with CCSS; and new teacher evaluation systems. Federal-state initiatives such as MSP can help address the challenge of implementing Common Core math as schools nationwide weather this perfect storm of reforms.

The Common Core State Standards for Mathematics and associated reforms in curriculum, professional development and assessment practices have potential to provide educators with new tools for teaching math and transform how students learn, stimulate their mathematical thinking and better prepare them for college and career success in a global society.

## Introduction

In this paper, we explore the importance of stimulating mathematical thinking and how new approaches to professional development, curriculum and assessment are helping states and districts achieve this paradigm change in mathematics education in the United States.

**The U. S. continues to lag behind many other nations on international math assessments** such as the Program for International Student Assessment (PISA) tests. The lack of progress in improving U. S. student math performance has serious implications for America's competitiveness in the global marketplace. Research by Hanushak and Woessmann<sup>1</sup> demonstrates a causal influence of PISA scores on Gross Domestic Product (GDP) in long-term participating PISA nations.

One key difference between the international PISA test and traditional high-stakes assessments in U. S. states is a focus in PISA on understanding and using math skillfully. As OECD (2013a) states in its analysis of U. S. 2012 PISA results, "one clear recommendation would be to focus much more on higher-order activities such as ... (understanding real world situations, transferring them into mathematical models, and interpreting mathematical results." (p. 74).

There is hope that implementation of new Common Core State Standards will turn U. S. performance around. Schmidt and Houng (2012)<sup>2</sup> analyzed the Common Core standards, finding them consistent with benchmarks derived from the standards of top-achieving nations, coherent and focused, and with the standards of U. S. states that are relatively high-achieving on international tests.

In making the transition to Common Core State Standards for Mathematics, teachers are called upon to make changes to their teaching and assessment approaches to stimulate mathematical thinking in their students and help them become mathematically skillful.

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Improving math teaching and learning has been a high federal priority for over a decade. Since its inception in 2002, the Mathematics and Science Partnerships (MSP) Program within the U. S. Department of Education (ED) has funded professional development activities designed to improve the knowledge of teachers and the performance of K-12 students in mathematics and science. MSP is part of the solution to the challenge of introducing new approaches to professional development, curriculum and assessment as Common Core math is implemented in many states.

The MSP program makes formula grants to states that make awards to partnerships comprised of local education agencies (LEAs), institutions of higher education and other entities. Each funded partnership must include at least one high-need LEA.<sup>3</sup>

*MSP is part of the solution to the challenge of introducing new approaches to professional development, curriculum and assessment as Common Core math is implemented in many states.*

<sup>1</sup> Hanushek, E. A., & Woessmann, L. (2010). *The high cost of low educational performance: the long-run impact of improving PISA outcomes*. Paris: Organization for Economic Cooperation and Development.

<sup>2</sup> Schmidt, W. H., & Houang, R. T. (2012, November). Curricular coherence and the Common Core State Standards for mathematics. *Educational Researcher*, 41(8), 294-308.

[http://gse.uci.edu/brownbags/Schmidt\\_Curricular%20Coherence%20and%20CCSSM\\_paper.pdf](http://gse.uci.edu/brownbags/Schmidt_Curricular%20Coherence%20and%20CCSSM_paper.pdf)

<sup>3</sup> <http://www2.ed.gov/programs/heatgp/faq.html> based on reaching a threshold percentage within at least one LEA school of (a) students in poverty, (b) less than highly qualified teachers, or (c) teacher turnover.

ED's MSP program goals include:

- 1) Improve the status and stature of mathematics and science teaching by encouraging institutions of higher education to improve mathematics and science teacher education
- 2) Focus on the education of mathematics and science teachers as a career-long process
- 3) Bring mathematics and science teachers together with scientists, mathematicians, and engineers to improve their teaching skills
- 4) Provide summer institutes and ongoing professional development for teachers to improve their knowledge and teaching skills<sup>4</sup>

*A large-scale STEM initiative proposed by the White House to replace MSP is stalled in Congress.*

A large-scale STEM initiative proposed by the White House to replace MSP is stalled in Congress. For FY2015, some components, such as STEM innovation networks, teacher pathways, and a master teacher corps, have been made separate priorities, alongside a renamed MSP program (Effective Teaching and Learning: STEM) funded at FY2012 levels.<sup>5</sup>

## Stimulating Mathematical Thinking: A Paradigm for Innovative Teaching

*Mathematical pedagogy is fundamentally concerned with engaging students in mathematical thinking and activity.* (Deborah Ball, PhD Dissertation, 1988<sup>6</sup>)

Devlin predicts that 100 years from now, the fundamental nature of math will still be a conceptual process based on patterns. But he believes that math will be used in ways we cannot predict today, such as helping people deal better with intractable challenges in human society.  
Teaching today's math

What is mathematics? Keith Devlin<sup>7</sup> of Stanford, also known as the "Math Guy" on National Public Radio, sees mathematics "as the science of patterns". Different math fields study different patterns. For example, geometry studies the patterns of shapes, and probability the patterns of chance. Devlin notes the fundamental shift by mathematicians in the mid-19<sup>th</sup> century from an emphasis on doing calculations, to an emphasis on conceptual thinking such as pattern making.

A special issue of *ASCD Express on Mathematical Thinking* (2013)<sup>8</sup> includes links to videos created in 1990 that show 3<sup>rd</sup> graders in the math class of teacher Deborah Ball. These students are engaged in deep discourse about the properties of even and odd numbers<sup>9 10</sup>.

<sup>4</sup> <http://www2.ed.gov/programs/mathsci>

<sup>5</sup> FY 2015 budget, pp. 20-21. <http://www2.ed.gov/about/overview/budget/budget15/summary/15summary.pdf>

<sup>6</sup> Ball, D. (1988). *Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education*, p. 172. Doctoral dissertation, Michigan State University. (AAT 8900008).

<sup>7</sup> Devlin, K. (2008). What will count as mathematics in 2100? In B. Gold & R. Simons (Eds.), *Proof and other dilemmas* (291-312). Portland, OR: Mathematical Association of America. Available: [http://www.stanford.edu/~kdevlin/Papers/Math\\_in\\_2100.pdf](http://www.stanford.edu/~kdevlin/Papers/Math_in_2100.pdf)

<sup>8</sup> <http://www.ascd.org/ascd-express/vol8/824-toc.aspx>

<sup>9</sup> University of Michigan. (2010a). *BetsyProof-Start*. Mathematics Teaching and Learning to Teach. Available: <http://hdl.handle.net/2027.42/65012>

### Video Excerpt

**Sean:** I was just thinking about six ... Because there can be three of something to make six, and three of something is like odd ... It could be an odd and an even number ...

**Keith:** That doesn't necessarily mean that six is odd.

**Teacher:** ... Do you remember from the other day the working definition we're using?

**Jeannie:** If you have a number that you can split up evenly ... it's an even number.

**Teacher:** Okay, so, Ofala, you're- you actually are suggesting [another] definition ...

**Ofala:** Well, an odd number is something that has one number left over.

**Teacher:** After you do what?

**Ofala:** After you circle the twos.

**Teacher:** Betsy, you think you could try one?

**Betsy:** I'll experiment with ... twenty one.

**Teacher:** What would you think twenty-one should be?

**Betsy:** Odd.

**Teacher:** Okay, everyone watch now and see if Betsy's experiment works out.

**Betsy:** [counting pairs] Four, five, six, seven, eight, nine ...

Project researchers noted that "In the course of this episode, the children practice mathematical reasoning, they develop mathematical arguments to justify their claims, and they learn to listen carefully to and evaluate the arguments of others." (p. 1).<sup>11</sup> In other words, these third graders are engaged in mathematical thinking with their teacher and classmates.

At the time of this study, Ball was a postdoctoral student at University of Michigan, where she now serves as Dean of Education. She continues to teach math to elementary students each summer, thereby remaining grounded in the strategies of effective teaching that she studied early in her career.

Ball (2008<sup>12</sup>) declares that "Good teaching is something to learn, not an inheritance" (p. 6). In 2013, she noted that "teaching all students to become mathematically skillful entails balancing three traditional dualities" (p. 9). To do so, K-12 teachers must:

1. Focus on mathematical concepts and procedures AND mathematical practices.
2. Teach in ways that are both teacher-directed AND student-centered.
3. Work to fill students' gaps AND to accelerate their capabilities with complex mathematical work. (p. 9)<sup>13</sup>

*"Good teaching is something to learn, not an inheritance"*

Twenty-five years after Ball's seminal research, professional development for math instruction is shifting to include a focus on stimulating mathematical thinking and building students' mathematical skillfulness.

<sup>10</sup> University of Michigan. (2010b). *SeanNumbers-Ofala; Transcript of SeanNumbers-Ofala*. Mathematics Teaching and Learning to Teach. Available: <http://hdl.handle.net/2027.42/65013>. Excerpts condensed for brevity.

<sup>11</sup> University of Michigan. (2010C). *Introduction to the SeanNumbers-Ofala Video*. Mathematics Teaching and Learning to Teach. Available: <http://hdl.handle.net/2027.42/65013>

<sup>12</sup> Ball, D. L. (2008). *Improving mathematics learning: Where are we and where do we need to head?* Paper presented for the Advanced Study Commission on Mathematics and Science Education, New York, NY, November 8, 2007.

<sup>13</sup> Ball, D. L. (2013, February). *Teaching all students to be mathematically skillful: Balancing the equation*. Scholastic Math Summit, Nashville, TN. Available: [http://www-personal.umich.edu/~dball/presentations/120112\\_NCCA.pdf](http://www-personal.umich.edu/~dball/presentations/120112_NCCA.pdf)

## Math Performance in Schools: International Trends

The most recent results from a 65-nation testing by the Organization for Economic Cooperation & Development (OECD) Program for International Student Assessment (PISA) highlight the challenge.

### **The United States continues to lag behind many other nations at the high school level.**

- In 2012, the average PISA math score of U. S. 15-year-olds (481) was below the OECD average (494), which represented a non-significant decline from PISA 2009 (487).
- The U. S. placed 26<sup>th</sup> out of 34 OECD member countries in math on PISA 2012, and 36<sup>th</sup> out of all 65 participating nations based on average scores in math.

### **Students in some nations continued to shine on PISA.**

- Fifteen-year-olds in Shanghai on average scored three grades higher than those in the U. S.
- Socio-economically disadvantaged students who lived in China, Korea or Japan were three times more likely than those who lived in the U. S. to show “resilience” by scoring in the top quarter of their nation’s test takers

### **Results from the 56-nation Trends in International Mathematics and Science Study (TIMSS) study in 2011 show that U. S. 4<sup>th</sup> graders did relatively well.**

In 2011, the average TIMSS math score of U.S. 4th-graders (541) was higher than the international average (500), and 12 score points higher than U. S. scores in 2007.

- U. S. 4th graders scored higher than those in 42 other nations, about the same in 6, and had lower scores than students in 8 other nations

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What accounts for the middling performance of U. S. fifteen-year-olds on international PISA tests, especially given the relatively better performance of U.S. 4<sup>th</sup> graders on TIMMS?

An OECD analysis of how American students performed on PISA test item types provides some insights.

*U.S. students have particular strengths in cognitively less-demanding mathematical skills and abilities, such as extracting single values from diagrams or handling well-structured formulae. And they have particular weaknesses in demanding skills and abilities, such as taking real world situations seriously, transferring them into mathematical terms and interpreting mathematical aspects in real world problems (OECD 2013, p. 74).*

This suggests that a learning focus on mathematical calculations and formula use may help U. S. students score relatively well in earlier grades, but not in later grades where test items become more complex. Based on these studies, there is a need to increase the skillfulness of U. S. students in their use and understanding of math.

OECD’s summary of U. S. PISA results in 2012 states that “An alignment study between the Common Core State Standards for Mathematics and PISA suggests that a successful implementation of the Common Core Standards would yield significant performance gains also in PISA”<sup>14</sup>.

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<sup>14</sup> United States Country note. PISA 2012. <http://www.oecd.org/pisa/keyfindings/PISA-2012-results-US.pdf>



## Mathematical Thinking and the Common Core

Adoption of the Common Core State Standards for Mathematics (CCSS-M) is underway in most parts of the U. S, ushering in new approaches to curriculum, professional development and assessment. CCSS-M practice and content standards are designed to students develop understanding and use math skillfully.

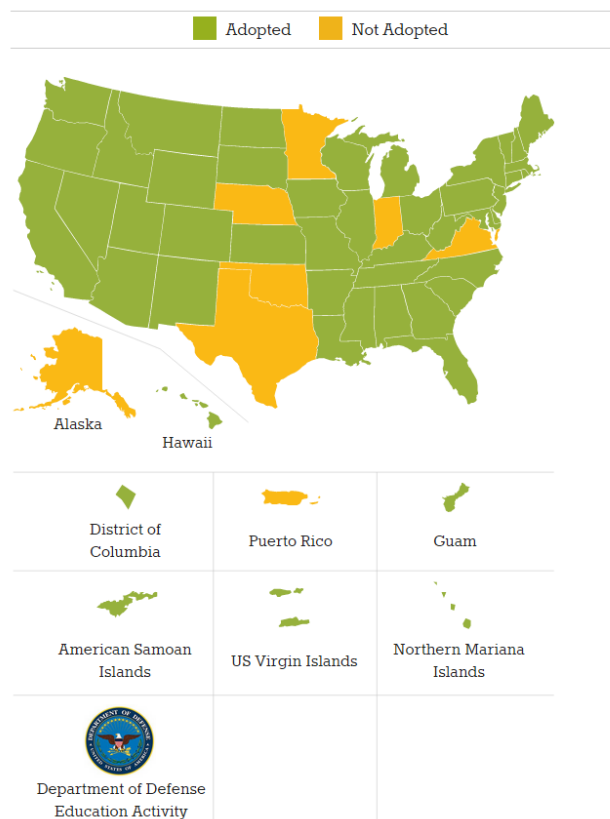
### Development and Implementation of the Common Core

A 2004 report by Achieve<sup>15</sup>, an organization developed by a bipartisan group of governors and business leaders in 1996, found that high school graduates lacked the knowledge and skills needed for success in college and career, and proposed development of a rigorous set of standards to address the problem.

In June 2009, the National Governors Association Center for Best Practices and the Council of Chief State School Officers (CCSSO) announced that they had convened a state-led Common Core Standards Initiative, in which 46 states and the District of Columbia had agreed to participate<sup>16</sup>. Participants supported the development of a common core of state standards in mathematics and English language arts for grades K-12, which would be “research and evidence-based, internationally benchmarked, aligned with college and work expectations and include rigorous content and skills<sup>17</sup>.”

As of December 2014, 43 states and the District of Columbia had adopted the Common Core State Standards (CCSS) in mathematics and/or English Language Arts (ELA)<sup>18</sup>. The *Common Core State Standards for Mathematics*<sup>19</sup> introduced eight standards for mathematical practice:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.



<sup>15</sup> Achieve. (2004). *Ready or not: creating a high school diploma that counts*. <http://www.achieve.org/ReadyorNot>

<sup>16</sup> [http://nga.org/cms/sites/NGA/home/news-room/news-releases/page\\_2009/col2-content/main-content-list/title\\_forty-nine-states-and-territories-join-common-core-standards-initiative.html](http://nga.org/cms/sites/NGA/home/news-room/news-releases/page_2009/col2-content/main-content-list/title_forty-nine-states-and-territories-join-common-core-standards-initiative.html)

<sup>17</sup> <http://www.corestandards.org/assets/Criteria.pdf>

<sup>18</sup> <http://www.corestandards.org/standards-in-your-state>

<sup>19</sup> [http://www.corestandards.org/assets/CCSSI\\_Math%20Standards.pdf](http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf)

These eight practices describe mathematical thinking skills that K-12 educators can use to help students develop understanding and use math skillfully as they progress along the math learning continuum. These practices are not new; they are primarily derived from NCTM's 2000 *Process Standards*<sup>20</sup> and the NRC's 2001 *Adding It up*<sup>21</sup>. However, they represent a change from traditional math practices. Under CCSS-M, more time is spent helping students attain a deeper understanding of mathematical content in fewer topics, beginning with basic operations in elementary school.

## Curriculum Development for Common Core Math

Progress is being made in overcoming challenges to implementing Common Core math curricula.<sup>22,23</sup>

### Signs of Progress

- A study of four Common Core “early implementer” school districts in 2013 found that they were struggling to provide high-quality CCSS-aligned and assessments, including textbooks and resources truly aligned with the new standards.

- In a spring 2014 survey of school districts in states that have adopted the Common Core for the Center on Education Policy, about 80% reported they had already begun teaching math and ELA curricula aligned to CCSS. However, only about 1 in 3 was already doing so in all schools. A majority were still in the process of developing or adopting aligned curriculum.

- Surprisingly, more than 80% of responding school districts in Common Core states reported they were developing their CCSS-aligned curricula locally.

One reason so many districts are developing their own curricula is the widespread availability of open-source resources (OER) for doing so. States are an important source. A recent national survey of state education agencies by CCSSO<sup>24</sup> identified 20 states that were undertaking OER initiatives by a variety of methods. CCSSO also hosts a K-12 OER Collaborative<sup>25</sup> launched in spring 2014, in which 11 states are currently creating and sharing Common Core aligned resources.

Another reason that districts are developing their own curricula may be a perceived lack of textbooks truly aligned with the CCSS standards. In February 2014<sup>26</sup>, Morgan Polikoff, a professor at University of Southern California, provided a first look at the results of his study of three mathematics textbooks for 4<sup>th</sup> graders in Florida that publishers claimed were aligned with Common Core math standards.

<sup>20</sup> National Council for the Teaching of Mathematics (2000). *Principles and standards for school mathematics. Process standards*. Available: <http://www.nctm.org/standards/content.aspx?id=322>

<sup>21</sup> National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Available: [http://www.nap.edu/openbook.php?record\\_id=9822&page=14](http://www.nap.edu/openbook.php?record_id=9822&page=14)

<sup>22</sup> Cristol, K. & Ramsey, B. s. (2014, February). *Common Core in the districts: an early look at early implementers*. Fordham Institute. <http://www.edexcellence.net/publications/common-core-in-the-districts>

<sup>23</sup> Rentner, D. S., & Kober, N. (2014, October). *Common Core State Standards in 2014: curriculum and professional development at the district level*. <http://www.cep-dc.org/displayDocument.cfm?DocumentID=441>

<sup>24</sup> CCSSO. (2014, November). State of the states: open educational resources in K-12 education. <http://www.ccsso.org/Documents/2014/Open%20Educational%20Resources%20in%20K-12%20Education-ver1.1.pdf>

<sup>25</sup> <http://k12oercollaborative.org>

<sup>26</sup> <http://rossier.usc.edu/polikoff-study-finds-textbooks-not-aligned-to-common-core-standards>

Polikoff's research team found three substantial alignment problems in each:

- They overemphasized memorization and procedures at the expense of conceptual skills included in the new Common Core math standards;
- Each had its own emphasis on specific content, while the standards do not;
- Each did not cover one-sixth to one-seventh of the standards.

In response to the findings of Polikoff and other researchers about the lack of alignment of many textbooks, a nonprofit, independent review service, EdReports.org<sup>27</sup> was launched in 2014 with funding from Helmsley Charitable Trust and the Gates and Hewlett foundations.

## Examples and Resources: Curriculum

EdReports.org reviews instructional materials for Common Core math alignment, usability, teacher support and differentiation, with a focus on helping teachers, principals, and district and state officials involved in purchasing make informed decisions. Reviews are conducted by teams of at least three trained expert educators who share results with a larger panel. EdReports.org reviews of K-8 math texts began in 2014. Reviews of grades 9-12 math texts begin spring 2015.

Common Core Implementation Tools<sup>28</sup>, a November 2013 guide prepared for Gates Foundation identifies 16 states with "homegrown" sets of open-source Common Core tools, often accompanied by crosswalks to prior standards, as well as 18 open-source national tools that have evidence of Common Core alignment. It also cites the 17 national tools most referenced by 45 states, noting that only one of them is not open-source. Illustrative Mathematics and Student Achievement Partners were most frequently referenced, with each cited by 36 states.

Illustrative Mathematics<sup>29</sup> is a source of freely available online resources that illustrate the math standards, including tasks, videos, lesson plans, curriculum modules, and other math content. It also hosts a blog<sup>30</sup> where new developments in Common Core tools are shared.

AchievetheCore.org, hosted by Student Achievement Partners, shares free, open-source resources to support Common Core implementation at the classroom, district, and state levels.

Common Core Toolbox<sup>31</sup>. The Dana Center for Mathematics at the University of Texas-Austin has compiled a set of tools that includes sample district curriculum frameworks.



<sup>27</sup> <http://edreports.org>

<sup>28</sup> Hanover Research. (2013, November). Final report. Common Core implementation tools.

(<http://s3.documentcloud.org/documents/1009965/commoncore-implementationtools-gates1113.pdf>)

<sup>29</sup> [www.illustrativemathematics.org](http://www.illustrativemathematics.org)

<sup>30</sup> <http://commoncoretools.me>

<sup>31</sup> <http://ccsstoobox.org>

## Professional Development for Effective Math Teaching

A September 2013 report for the Center for Public Education<sup>32</sup> reviewed the role of professional development in the Common Core, finding it key to the reform's success. As Common Core standards focus on teaching for critical thinking, "professional development needs to emphasize practices that will turn students into critical thinkers and problem solvers (p. 3)."

Case studies of four Common Core "early implementer" districts in 2013<sup>33</sup> revealed that they were struggling to provide high-quality CCSS-aligned professional development content, and did not have enough instructional coaches and master teachers highly qualified in CCSS to help them implement the new standards district-wide.

In a spring 2014 survey<sup>34</sup>, about two thirds of districts in Common Core states said all or practically all teachers had received CCSS-related professional development, but only a third felt teachers were adequately prepared to teach the Common Core during the 2013-14 school year. District teachers were providing much of the CCSS-related professional development.

As districts rapidly implement the Common Core, one source of models of effective professional development models is U. S. Department of Education's MSP program, which has provided valuable graduate education and continuing professional development for thousands of math teachers since it was established under the No Child Left Behind Act of 2001, the last reauthorization of the federal Elementary and Secondary Education Act.

A recent annual report by ABT Associates<sup>35</sup> shows that in FY2012, over 40,000 K-12 educators participated in MSP. These educators in turn taught over 2.3 million students. Funded MSP partnership grants within U. S. state-level MSP programs varied in the professional development (PD) models they followed, and in program duration.

### *State-funded MSP grants (2012)*

- 53% conducted summer institutes with school-year follow-up activities. These programs were longer in average duration (97 hours).
- 47% offered a broad range of types of professional development onsite and online, graduate courses, and professional learning communities, usually in combination with a shorter summer component. These mixed programs were on average shorter in duration (68 hours).

Professional development programs in funded MSP grants may also be classified as being more district-based or more university-based. For example, in Illinois:

- Following a district-based institute model, the entire math faculty of a mid-sized high school participated in an MSP grant. They agreed on a curriculum redesign model, and each teacher took on redesign of one or more courses in an open source LMS during the institute. A core group of teachers conducted most staff development activities. A university partner provided curriculum planning assistance and professional development on special topics. Teachers used shared planning time to continue their professional learning after the summer institute.
- Following a university-based model, one MSP grantee

<sup>32</sup> Gulamhussein, A.. Teaching the teachers. <http://www.centerforpubliceducation.org/teachingtheteachers>

<sup>33</sup> Cristol & Ramsey, 2014.

<sup>34</sup> Rentner & Kober, 2014.

<sup>35</sup> Abt Associates. (2013, November). *Mathematics and science partnerships: Summary of performance period 2011 annual reports*. [http://www.ed-msp.net/images/public\\_documents/document/annual/MSP%20PP11%20Annual%20Final%20Report.pdf](http://www.ed-msp.net/images/public_documents/document/annual/MSP%20PP11%20Annual%20Final%20Report.pdf)

adopted a research-based curriculum product and offered extensive university-based professional development and technical support, while another offered a university Master's degree focused on teacher leadership.

- While the more university-based programs were strong and cohesive and demonstrated many effective professional learning strategies, such programs are not as scaleable—many districts cannot rely upon a nearby university for expertise or afford an expensive outside curriculum.

Oyer, Jarosewich, Greaney, de la Torre, and Downey (2014<sup>36</sup>) reported that from 2008 through 2012, State of Illinois-funded MSP professional development projects improved teacher knowledge, regardless of whether a master's degree program or workshop institute model was followed by the grantee.

Math and science teachers who participated in MSP grants nationwide in FY2012 showed gains on pre-and post-assessments of subject matter content knowledge, as did their students.

Ball, Thames and Phelps (2008)<sup>37</sup> found that both subject matter and pedagogical content knowledge were needed to teach mathematics effectively. Like subject matter knowledge, pedagogical content knowledge for Common Core teaching is an important focus for aligned professional development.

In FY2013<sup>38</sup>, Illinois MSP projects all provided professional development on the Common Core standards and their implementation in the classroom. On end of year surveys:

- Illinois MSP teachers reported gains in satisfaction with their math content knowledge (81 % to 91%), use of Common Core math practices (87% to 93%) and math instruction (77% to 85%).
- Almost all teachers (97%) were confident in FY2013 that they would continue to integrate the Common Core Mathematical Practices promoted by the grant, but fewer were confident about having access to Common Core materials or district support (83%).<sup>39</sup>
- Students were most likely to report classroom use in FY2013 of elements of the Mathematical Practices such as breaking down complicated problems into smaller parts (65%) and finding repeated calculations to make their solutions easier (59%).

#### *National Gains in Teacher Knowledge (2012)*

- Participating teachers assessed as part of MSP evaluations showed significant gains in content knowledge in math (63%), slightly more than in FY2011 (61%).
- Students of participating teachers scored at the proficient level or above in content knowledge in math (55%) a considerable drop from FY2011 (64%). Part of the reason might be the changeover to Common Core math instruction, which has been related to initial drops in state assessment scores (see *Mathematical Thinking and New Assessments of Learning*).

<sup>36</sup> Oyer, E., Jarosewich, T., Greaney, D., de la Torre, J., & Downey, G. (2014, April). *Models of transformative collaboration: Effectiveness of university-industry-school partnerships in graduate versus workshop models of professional development*. Paper presented at the 2014 annual meeting of the American Educational Research Association, Philadelphia, PA.

<sup>37</sup> Ball, D.L., Thames, M. H., & Phelps, G. (2008) Content knowledge for teaching: what makes it special? *Journal of Teacher Education* (59)5, 389-407.

<sup>38</sup> Abt Associates. (2014, December). *Mathematics and science partnerships: Summary of performance period 2012 annual reports*. [http://www.ed-msp.net/images/public\\_documents/document/annual/MSP%20PP12%20Annual%20Final%20Report%2012-1-14%20FINAL.pdf](http://www.ed-msp.net/images/public_documents/document/annual/MSP%20PP12%20Annual%20Final%20Report%2012-1-14%20FINAL.pdf)

<sup>39</sup> Oyer, E. O. (2013, December). *IMSP WIP4 year 2 report*. Carmel, IN: EvalSolutions.

## Examples and Resources: Professional Development

Many states that have Common Core toolkits offer at least some free professional development resources online.

- CCSS Professional Learning Modules<sup>40</sup> is an online professional development series offered by California Department of Education on its Digital Chalkboard Web site.
- EngageNY provides online Professional Development Kits<sup>41</sup> that include presentations, a facilitator's guide and a CCSS alignment for each professional development activity.
- Student Achievement Partners offers a series of free math professional development resources at AchievetheCore.org, called the Complete Collection of PD Modules and Courses<sup>42</sup>. The resources include presentations, videos, facilitator instructions, and hands-on activities, and are ready for use in a group or individual setting.
- The NDSL Math Common Core Collection<sup>43</sup> includes thousands of free online digital learning objects that can be searched by education level, resource type, and Common Core Math standards.
- The Council of Chief State School Officers has developed a CCSSO Math Tool<sup>44</sup> to enhance state implementation plans for Common Core math instruction. It focuses on what teachers need to know and be able to do, how states can support teachers, and how to foster collaborations for sustainability. It includes illustrative examples and research evidence for each key strategy cited.



MSP program initiatives around the nation provide valuable examples of effective professional development models and strategies. For example:

- *Chandler Intel Mathematics Academy*, an MSP partnership with Central Arizona College and Arizona State University and Intel Math. Teachers joined mathematics learning communities to improve their mathematical thinking and practices and increase use of formative assessment and differentiation. Implementation of a school-wide model was required for participation. The 140-hour training involved a summer institute and workshops during semester breaks.
- *Aurora Partners for Leadership in Teaching (APLET)*, a district-university partnership that provided 75 teachers with the opportunity to earn an integrated Master's degree program in teacher leadership in high school and middle school mathematics, including 24 credit hours in math content and 12 in teacher leadership. Participants integrated software in their classrooms, conducted action research, interacted with professionals, and completed a field experience that connected the classroom with authentic math and science in professional practice.<sup>45</sup>
- *Nebraska Math*, a large-scale, statewide project focused on transitions in math learning such as the algebra transition from middle to high school, while supporting the transition of new University of Nebraska secondary math teachers to the classroom. The five-year project impacted about 900 K-12 teachers and 60,000 students. A follow-up foundation grant in 2013 allowed 200 Omaha teachers to participate in continued training via a district academy program.

<sup>40</sup> <http://www.cde.ca.gov/re/cc/ccssplm.asp>

<sup>41</sup> <https://www.engageny.org/resource/professional-development-teachers>

<sup>42</sup> <http://achievethecore.org/page/680/professional-development-list-page>

<sup>43</sup> <https://nsdl.oercommons.org>

<sup>44</sup> <http://mathtool.ccsso.org>

<sup>45</sup> Abt Associates. (2012, November). *Mathematics and science partnerships: Summary of performance period 2010 annual reports*. [http://www.edmsp.net/public\\_documents/document/annual/MSP%20PP10%20Annual%20Final%20Report.pdf](http://www.edmsp.net/public_documents/document/annual/MSP%20PP10%20Annual%20Final%20Report.pdf)

# New Assessments of Learning

New assessments in learning are integrally linked to the rollout of Common Core State Standards in Math (CCSS-M). New approaches to formative assessments are being tested, and new state-level systems of assessment of student math performance aligned with CCSS-M are currently being rolled out in many states. These new approaches to formative and summative assessment all incorporate mathematical thinking as a key strategy for student learning.

Kevin Welner of the National Education Policy Center has expressed concerns about whether formative and summative assessment strategies necessary to close “opportunity gaps” would be included in CCSS implementation<sup>46</sup>. He fears that the new common standards and assessments will simply become a new driver for accountability-focused politics and market-based privatization policies.

- To be truly beneficial, he said, **the Common Core needs to be combined with “deep, challenging assessments [within] a feedback process to provide supports and resources to students and their teachers ... [and] inform teaching and curriculum (p.1)”**.

The 18-state CCSSO working group known as FAST SCASS (Formative Assessment for Students and Teachers, State Collaborative on Assessment of Student Standards) had defined formative evaluation as: “a process used by teachers and students during instruction that **provides feedback to adjust ongoing teaching and learning** to improve students’ achievement of intended instructional outcomes.”<sup>47</sup>

A report for ETS (2012)<sup>48</sup> sees professional development as critical to bringing formative assessment into classroom practice. Research by ETS researchers has found that teachers can adopt formative assessment, if provided with “information, structures, support, and sufficient time” (p. 7).

- Wylie & Ciofalo (2008)<sup>49</sup> explored the use of formative evaluation questions related to common misconceptions in math and science classes to stimulate student discussion and monitor understanding. They found that this technique provided important evidence about learning progress to teachers and students, while helping some students overcome misconceptions.
- Arieli-Attali, Wylie & Bauer (2012)<sup>50</sup> conducted a study in which middle school math teachers first administered an online formative test to see where students stood on a specific learning progression then intervened with incremental learning tasks focused on learning progression.

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<sup>46</sup> Welner, K. G. (2014). The lost opportunity of the Common Core State Standards. *Education Week*, March 28, 2014. Available: [http://www.edweek.org/ew/articles/2014/04/01/kappan\\_welner.html](http://www.edweek.org/ew/articles/2014/04/01/kappan_welner.html)

<sup>47</sup> CCSSO. (2012). *Distinguishing formative assessment from other educational assessment labels*. <http://www.ccsso.org/Documents/FASTLabels.pdf>

<sup>48</sup> Wylie, E. C., & Lyon, C. (2012). Formative assessment: supporting students’ learning. [http://www.ets.org/research/policy\\_research\\_reports/publications/periodical/2012/ifif](http://www.ets.org/research/policy_research_reports/publications/periodical/2012/ifif)

<sup>49</sup> Wylie, E. C. & Ciofalo, J. F. (2008, September 5). Supporting teachers’ use of individual diagnostic items. *Teachers College Record*.

<http://www.serve.org/uploads/docs/Events%20&%20Webinars/Wylie%20and%20Ciofalo,%20TCR%202008.pdf>

<sup>50</sup> Arieli-Attali, M., Wylie, C., & Bauer, B. (2012). *The use of three learning progressions in supporting formative assessment in middle school mathematics*. Paper presented at the annual meeting of American Educational Research Association, Vancouver, BC.

- ETS researchers are exploring new game-based approaches to formative assessment (see Examples and Resources). They have also worked with FAST SCASS states to field test a FAST observation protocol for peer coaching and teacher self-evaluation, and online professional development modules based on FAST.<sup>51</sup>

## State Consortia Assessments Aligned with the Common Core

### Example

*In the 2010-2012 Illinois MSP grant round, grantees were encouraged to incorporate such student performance products into teacher professional development activities.*

*Six Illinois projects elected to participate, with students of participating teachers asked to incorporate performance tasks into classroom implementation of grant-developed curricula. As part of their professional learning, teachers reviewed their student's products across eleven indicators in five areas.*

- *Most student product assignments (89%) were collaborative in nature. Over half involved real world concepts, and about a third involved skills application.*
- *Teachers rated student products as proficient or excellent most frequently for Product Development and Learning Standards (both 92%) and for Information Literacy (89%).*
- *Teachers rated student products as proficient or excellent somewhat less frequently for Higher Order Thinking, Problem-Solving, and Decision Making (73%), and Digital Citizenship (53%).*

In 2010, two state-led assessment consortia separately received four-year grants through the U. S. Department of Education's Race to the Top program, for the purpose of developing a common set of K-12 assessments in mathematics and ELA tied to college and career readiness.

- The District of Columbia and 14 states belong to the Partnership for Assessment of Readiness for College and Careers (PARCC<sup>52</sup>).
- The Smarter Balanced Assessment Consortium, or Smarter Balanced, includes 21 states.
- Operational rollout for both state assessment systems is underway in 2014-15.

Like PISA, the PARCC and SBAC assessments focus on deeper learning. Both test students' ability to read complex text, complete research projects, complete speaking and listening assignments, within an online environment. According to a RAND study, "The Smarter Balanced and PARCC assessment designs reflect a realization of the importance of integrating formative and summative assessments<sup>53</sup>". These assessments will replace existing high-stakes summative assessment systems in adopting states.

The incorporation of student performance products has been explored as a way of introducing formative assessment into math instruction. Such products have the potential to stimulate student mathematical thinking and use of mathematical practices.<sup>54</sup>

<sup>51</sup> Wylie, E.C., Lyon, C. (2013, May). *Using the formative assessment rubrics, reflection and observation tools to support professional reflection on practice.* <http://community.ksde.org/LinkClick.aspx?fileticket=qr-uWOo88T4%3d&tabid=5803&mid=14222>

<sup>52</sup> <https://www.parcconline.org>

<sup>53</sup> Faxon-Mills, S., Hamilton, L. SI, Rudnick, M., & Stecher, B. M. (2013). New assessments, better instruction? [http://www.rand.org/pubs/research\\_reports/RR354.html](http://www.rand.org/pubs/research_reports/RR354.html)

<sup>54</sup> Heitlin, L. (2014, Sept. 24). Will Common Core testing platforms impede math tasks? *Education Week.* <http://www.edweek.org/ew/articles/2014/09/24/05math.h34.html>



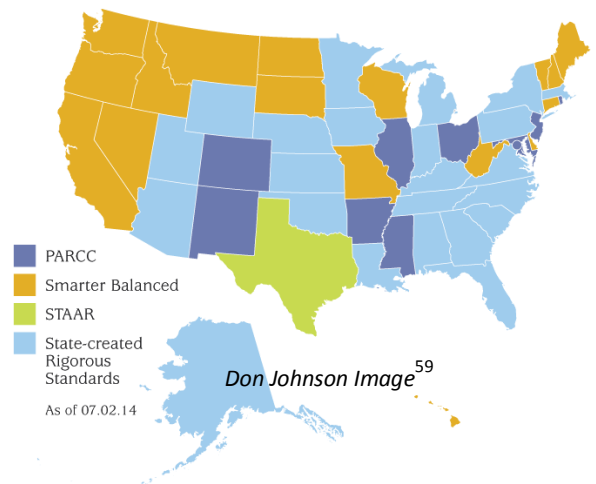
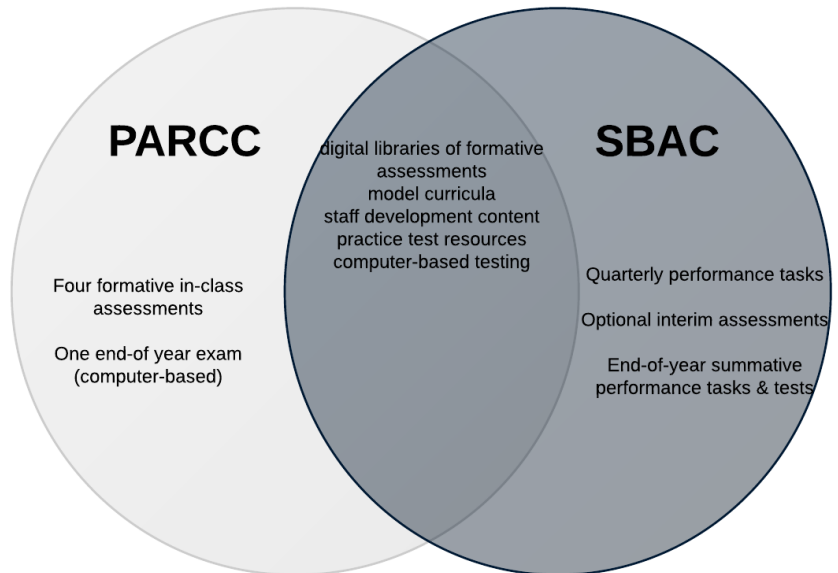
Figure 1. Higbee (2012) provides a useful comparison of PARCC & SBAC systems.

Of the 35 states involved in one of the assessment consortia, about 26 currently plan to implement the new tests in 2014-15. Some of the others have decided to incorporate individual items into their own state assessments<sup>55</sup>.

- A few states have dropped out of each consortium, and some never joined either one<sup>56</sup>. These states have expressed concerns about high costs, a lack of necessary technology in schools, negative public perceptions, and drops in test scores.
- New concerns have been raised recently about whether open-ended math assessment items in the online PARCC and Smarter Balanced assessments can accurately assess student solution work, as students' answers can be entered in the system<sup>57</sup>, but not their solution work. The consortia are working to address these concerns.

An October 2014 report<sup>58</sup> presents the results of a survey of a representative sample of districts in states nationwide that are implementing Common Core assessments.

- A majority of districts appeared to reserve judgment, saying it was "too soon to tell" whether the tests would be an improvement over their states' existing assessments.
- Most plan to keep in place local interim and formative assessments, but about half plan to revise them.
- A majority reported they faced challenges in terms of the technology and expertise needed to implement the consortia assessments online.



<sup>55</sup> Heitlin, L. (2014, August 20). Common Core textbooks to receive online ratings. *Education Week*. 34(1), p. 1, 18.

<sup>56</sup> <http://truthinamericaneducation.com/common-core-assessments/what-states-have-pulled-out-of-their-common-core-assessment-consortium/>

<sup>57</sup> Heitlin, L. (2014, Sept. 24). Will Common Core testing platforms impede math tasks? *Education Week*. <http://www.edweek.org/ew/articles/2014/09/24/05math.h34.html>

<sup>58</sup> Rentner, D. S., & Kober, N. (2014, October). *Common Core State Standards in 2014: district implementation of consortia-developed assessments*. <http://www.cep-dc.org/displayDocument.cfm?DocumentID=442>

<sup>59</sup> <http://donjohnston.com/testing-accommodations/#.VJH89ivF98E>

## State Teacher and Administrator Assessment Systems

At the same time, states have been implementing assessment systems intended to help assure teacher and administrator quality, as required under No Child Left Behind.

### Examples and Resources: Assessment

*In 2013, 28 states required annual teacher evaluations, 41 required them to include measures of student learning, and 19 required effectiveness to be factored into teacher tenure decisions. Teacher and administrator evaluation reforms may make the transition to Common Core math and new assessments more challenging for teachers and schools.*



The Mathematics Assessment Project<sup>60</sup> (MAP) offers free online assessment resources, including classroom challenge lessons for teacher use in formative assessment, summative student

performance tasks and test items, and professional development for formative assessment.

The Mathematics Design Collaborative (MDC), parent organization of MAP, offers course outlines and a video library to support the classroom challenge on its College Ready<sup>61</sup> site.

The ETS Assessment Games Challenge<sup>62</sup> challenges educators to create a game that acts as a formative math assessment, while incorporating one of two learning progressions ETS developed through its research. The winning games can be browsed, played and rated online.<sup>63</sup>

Raising the Bar: Becoming Assessment Ready<sup>64</sup> is a guide developed in February 2014 by the Coalition for School Networking. It provides detailed information about the online testing implementation timelines of PARCC and Smarter Balanced, and what district-level IT managers need to know to plan for effective implementation of these tests in their schools.

## A Call to Action

The United States continues to lag behind many other nations in math learning at the high school level. The new Common Core standards and assessments have strong potential to improve math teaching and learning in the U. S., and the nation's global standing and leadership in this area.

The MSP program, now called Effective Teaching and Learning: STEM, remains the key state-federal initiative with the potential to improve mathematics education. Its focus is on providing effective professional development opportunities for math and science teachers nationwide, including professional development that supports implementation of Common Core math. Whatever it is called, this program needs to be continued and strengthened.

State-level incorporation of the Common Core math standards into MSP professional development activities will help ensure that teachers incorporated mathematical thinking into instruction, so that students become mathematically skillful. The same can be said of incorporation of the Next Generation Science Standards to help students become scientifically skillful. Together, these standards support the larger vision of effective STEM education in the U. S., building STEM careers and global competitiveness.

<sup>60</sup> <http://map.mathshell.org>

<sup>61</sup> <http://collegeready.gatesfoundation.org/Learning/MathDesignCollaborative>

<sup>62</sup> <http://etsgameschallenge.challengepost.com/details/learningprogression>

<sup>63</sup> <http://etsgameschallenge.challengepost.com/submissions/9562-equations-squared>

<sup>64</sup> <http://www.ena.com/wp-content/uploads/2014/02/Raising-the-BAR-WP.pdf>

## Supporting Common Core Math Instruction and Student Mathematical Thinking

By effectively implementing Common Core math curriculum, professional development, and assessment systems, states and districts are supporting math teaching that stimulates mathematical thinking and help students become mathematically skillful. Examples of this work in progress include:

<b>Curriculum Development</b>	<b>Professional Development</b>	<b>Assessment</b>
<ul style="list-style-type: none"> <li>• Independent national nonprofits and state education agencies are assisting districts with reviewing instructional content for alignment with the Common Core in Math.</li> <li>• Many districts are using state and national open-source resources to develop Common Core math curriculum frameworks and their own CCSS-aligned curricula.</li> <li>• A number of Common Core adopting states have developed online toolsets to assist districts in math curriculum development, referencing additional tools developed by national providers.</li> <li>• National nonprofit providers are offering a wide range of open-source Common Core resources that are being adopted in states that have not developed their own “homegrown” toolsets.</li> <li>• Some states offer repositories of Common Core and state-aligned curriculum units developed by master educators, along with well-aligned units submitted by teachers in the field.</li> <li>• Some national providers offer digital learning objects which teachers can embed in gateway courses to address key student misconceptions or to enrich math curriculum.</li> </ul>	<ul style="list-style-type: none"> <li>• Nationwide, district teachers are providing much Common Core math professional development, with some professional development provided by universities and other partners.</li> <li>• MSP grant teachers are receiving professional development that increases both the depth of their math content knowledge and their pedagogical content knowledge, including training in Common Core math practices and opportunities to practice their integration in the classroom.</li> <li>• Professional and curriculum development programs that primarily build internal district capacity may be more scaleable nationwide than those that primarily rely on external partners.</li> <li>• Evidence of changes in instructional practice may be incremental for many MSP grant teachers, involving limited adoption of questioning techniques and some mathematical practices.</li> <li>• Intensive, long –term professional development that includes institutes and year-round professional learning appears most effective in achieving changes in teacher practice.</li> </ul>	<ul style="list-style-type: none"> <li>• Adopting either the PARCC or Smarter Balanced assessment system to replace their traditional high-stakes assessment systems (about half of states). Both systems combine formative and summative assessment to better inform Common Core teaching and learning.</li> <li>• Incorporating Common Core assessment items into existing assessments or delaying changes to assessments (some states; <b>a limited number have moved away from the Common Core</b>).</li> <li>• Guidance is available for states and districts in making the transition to online testing under the Common Core assessments, but many districts do not yet have the needed technology in place. <b>Concerns have been raised about whether online testing platforms impede student math work</b>.</li> <li>• The incorporation of Common Core aligned student performance tasks and formative assessment activities in professional development and classroom integration has been explored as a way to stimulate mathematical thinking and use of mathematical practices, while giving students the opportunity to practice for open-ended problems in Common Core assessments.</li> <li>• National providers offer a variety of free online tools that teachers can use in formative assessment, including lessons, performance tasks and test items, and professional development.</li> <li>• New value-added approaches to teacher and administrator assessment are also being rolled out nationwide, in response to No Child Left Behind mandates.</li> </ul>

Several reforms are being implemented in the same timeframe in many U. S. states: the Common Core State Standards (CCSS) and associated curriculum, instructional and assessment reforms; the PARCC/Smarter Balanced common sets of state assessments aligned with CCSS; and new teacher and principal evaluation systems.

This perfect storm of reforms may, at least in the short term, impact the ability of K-12 mathematics teachers to focus on meaningful incorporation of new ways of teaching math in their classroom practice. While we have heard this “too many reforms” concern from math teachers and school administrators as we visit schools, we have also heard their support for the Common Core State Standards as an innovation they hope will not be another “flash in the pan”.

Many teachers have experienced multiple waves of reform in their districts and changes in state teaching and learning standards that necessitate changes in how they plan and implement instruction. However, only some of what teachers learn through professional development related to these transient reforms actually impacts their teaching practice long-term.

We hope that Common Core math, with its focus on stimulating mathematical thinking and practices in math classrooms, is one of the new ways of doing things in education that “sticks”. It has the potential to provide educators with new tools for teaching, and to transform how students learn, better preparing them for college and career success in a global society.

